

## Semester - III

### ADVANCED GRAPH THEORY---I

*Course No. MM-CP-308*

*Duration of Examination: 3 hrs*

*Maximum Marks: 100*

*(a) External Exam: 80*

*(b) Internal Exam: 20*

#### Unit I

Graph matrices, Incidence matrix  $A(G)$ , Two graphs are isomorphic iff their incidence matrices differ only by permutation of rows and columns, Rank of incidence matrix, the rank of incidence matrix of connected graph with  $n$  vertices is  $n-1$ , sub matrices of  $A(G)$ , Cut-set matrix  $C(G)$ , Rank  $C(G) = \text{rank of } A(G) = \text{rank of } G$ , Relationships between  $A_j$ ,  $B_j$ ,  $C_j$ , Path matrix, Adjacency matrix  $X(G)$ , Powers of  $X$ , Relation between  $A(G)$  and  $X(G)$ .

#### Unit II

Coloring, chromatic number  $\chi(G)$ , A graph is bicolorable iff it has no odd cycles, Bounds for  $\chi(G)$ , Bounds on sum and product of the chromatic number of a graph and its complement, Five color theorem, Four color theorem (statement only), Every planar graph is four colorable iff every cubic bridgeless plane map is 4-colorable, Every planar graph is 4-colorable iff  $\chi'(G) = 3$  for every bridgeless planar graph, Heawood Map-coloring theorem, Uniquely colorable graphs.

#### Unit III

Edge graphs, A connected graph is isomorphic to its edge graph iff it is a cycle, Whitney's theorem on edge graphs, Characterization of edge graphs of trees, edge graphs and traversibility, total graphs, Eccentricity sequence and sets, Lesniak theorem for trees, Construction of trees, Neighborhoods, Lesniak theorem for graphs.

#### Unit IV

Digraphs, types of digraphs, Digraphs and binary relations, Directed paths and connectedness, Euler digraphs, Trees with directed edges, Arborescence, Ordered trees, Spanning arborescence, Fundamental cycles in digraphs, Matrices  $A$ ,  $B$ ,  $C$  of digraphs, The determinant of every square sub matrix of  $A$  is 1, -1 or 0. Rows of cycle matrix are orthogonal to the rows of the incidence matrix, Number of spanning trees, Fundamental cycle matrix, Adjacency matrix of a digraph, Connectedness and the adjacency matrix, Number of arborescence, Tournaments, Score sequences, Landau's theorem, Oriented graphs.

#### References

1. F. Harary, Graph Theory, Addison- Wisley.
2. Narsingh Deo : Graph Theory with applications to Engineering and Computer Sciences, P-III.
3. F. Harary, F.R. z. Norman and D. Cartwright ; Structure Models : An Introduction to the theory of Directed graphs, J. wiley.
4. K.R Parthasarty : Basic Graph Theory, Tata Mc-Graw Hill
5. B. Bollobas : Graphs theory, Acad, Press London.
6. D.B. West Introduction to Graph Theory Prentice,
7. T.L.Saaty and P.C. Kainen : The Four Color Problem, Dover Pub.
8. S.Pirzada and A.Dharwadkar, Graph Theory, Universities Press(Orient Longman)