M.A/M.Sc Mathematics Semester 4th

Effective from academic session 2011 ⟹ Repetition for 2012 with minor change

DIFFERENTIAL GEOMETRY

Course No. MM-CP-402

Unit I

Curves : Differentiable curves, Regular point, Parameterization of curves, arc-length, and arc-length is independent of parameterization, unit speed curves. Plane curves: Curvature of plane curves, osculating circle, centre of curvature. Computation of curvature of plane curves. Directed curvature, Examples: Straight line, circle, ellipse, tractrix, evolutes and involutes. Space curves: Tangent vector, unit normal vector and unit binormal vector to a space curve. Curvature and torsion of a space curve. The Frenet-Serret theorem. First Fundamental theorem of space curves. Intrinsic equation of a curve. Computation of curvature and torsion. Characterization of Helices and curves on sphere in terms of their curvature and torsion. Evolutes and involutes of space curves.

Unit II

Surfaces; Regular surfaces with examples, coordinate charts or curvilinear coordinates, change of coordinates, tangent plane at a regular point, normal to the surface, orient able surface, differentiable mapping between regular surfaces and their differential. Fundamental form or a metric of a surface, line element, invariance of a line element under change of coordinates, angle between two curves, condition of orthogonality of coordinate curves. Area of bounded region, invariance of area under change of coordinates.

Unit III

Curvature of a Surface: Normal curvature, Euler's work on principal curvature,. Qualitative behavior of a surface near a point with prescribed principal curvatures. The Gauss map and its differential. The differential of Gauss is self-adjoint. Second fundamental form. Normal curvature interms of second fundamental form. Meunier theorem. Gaussian curvature, Weingarten equation. Gaussian curvature $K(p)= (eg-f^2)/EG-F^2$ .surface of revolution. Surfaces with constant positive or negative Gaussian curvature. Gaussian curvature in terms of area. Line of curvature, Rodrigue's formula for line of curvature, Equivalence of Surfaces: Isometry between surfaces, local isometry, and characterization of local isometry.

Unit IV.

Christoffel symbols. Expressing Christoffel symbols in terms of metric coefficients and their derivative. Theorema egregium (Gaussian curvature is intrinsic). Isometric surfaces have same Gaussian curvatures at corresponding points. Gauss equations and Manardi Codazzi equations for surfaces. Fundamental theorem for regular surface. (Statement only).

Geodesics: Geodesic curvature, Geodesic curvature is intrinsic, Equations of Geodesic, Geodesic on sphere and pseudo sphere. Geodesic as distance minimizing curves. Gauss-Bonnet theorem (statement only).Geodesic triangle on sphere. Implication of Gauss-Bonnet theorem for Geodesic triangle.

Recommended Books:

- John Mc Cleary: Geometry from a differentiable Viewpoint. (Cambridge Univ. Press)

Suggested Readings:

1. W. Klingenberg: A course in Differential Geometry (Spring Verlag)
2. C.E. Weatherburn: Differential Geometry of Three dimensions.
3. T. Willmore : An Introduction to Differential Geometry
4. J. M. Lee : Riemannian Manifolds, An Introduction to Curvature (Spring