Government Degree College, Baramulla (Autonomous)

Term End External Examination 4th Semester (Session- July 2024)

Subject: Mathematics

Course No and Title: MMTC3422M/Theory of Numbers

Time: 2.15 hours **Max Marks:100** **Min. Marks:40**

Section A: Objective Type Questions

Q1. Choose the appropriate Answer: (8x1.5=12)

i. Congruence is a

A Reflexive relation **B** Symmetric relation
C Transitive relation **D** All the above

ii. Which of the following does not belongs to Reduced Residue System (mod7)

A 1 **B** 7
C 3 **D** 5

iii. Which of the following linear Diophantine equation(s) has (have) integral solution

A $4x+6y=8$ **B** $3x+6y=7$
C $2x+4y=7$ **D** $5x+10y=8$

iv. If $\gcd(a,b)=d$ then $\gcd\left(\frac{a}{d},\frac{b}{d}\right)=$

A a **B** b
C d **D** 1

v. The order of 2 modulo 7 is

A 1 **B** 2
C 3 **D** 4

vi. The smallest primitive root of 17 is

A 3 **B** 5
C 6 **D** 7

vii. Consider the cubic equation $Z^3+3HZ+G=0$. The equation has one real and two imaginary roots when its discriminant is

A less than zero **B** Greater than zero
C Equal to zero **D** Does not depend on discriminant

viii. The number of negative roots of $f(x)$cannot exceed the number of changes of signs in

A $f(x)$ **B** $f(x)$ and $f(-x)$
C $f(-x)$ **D** $f(x)$ or $f(-x)$

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Section-B: Descriptive Type Questions (Short Type)

Q2: Answer all the Questions (8 x 4 =32)

i. Prove that the product of any three consecutive integers is divisible by $3!$.

ii. Show that every integer $n>1$ can be expressed as a product of one or more primes.

iii. Define Complete Residue System (CRS) and write CRS (mod 5) in two different ways.

iv. State and prove Quadratic reciprocity law.

v. Solve the equation $x^4-14x^3+73x^2-168x+144=0$. It being given that it has two pairs of equal roots.

vi. Divide $x^4-10x^3+35x^2-48x+27$ by $(x-1)(x-2)(x-3)$ and hence find quotient and remainder.

vii. Solve $x^3+x+10=0$ by Cardan's method.

viii. If $\alpha,\ \beta,\ \gamma$ be the roots of $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\alpha^2\beta^2}$.

Section – C: Descriptive Type Questions (Medium Type)

Answer all the questions: (4 x 7=28)

Q 3. State and prove the necessary and sufficient condition for solvability of linear Diophantine equations.

OR

State and prove Euclid's division algorithm.

Q 4. State Goldbach Conjecture and write down at least five integers that satisfy the conjecture. Also prove there are infinite number of primes.

OR

Prove that there are infinitely many primes of the form $6q+5$. Also prove that if $(a,b)=1$ then there are infinitely many primes of the form $aq+b$.

Q 5. Show that the integer 2^n has no primitive root for $n\geq 3$.

OR

State and prove the Euler's Criterion for an integer a to be a quadratic residue (mod p), where p is an odd prime.

Q6. Write down the Cardan's method of solution of cubic equations.

OR

Write down the Descartes method of solution of biquadratic equations.

Section – D: Descriptive Type Questions (Long Type)

Answer any two of the following: **(2 x 14=28)**

Q 7. Define number theoretic function and show that τ and σ are multiplicative. Also define Mobius function and state and prove Mobius Inversion Formula.

8. State and prove Chinese Remainder Theorem. Also write down at least one application of the Theorem.

Q 9. State Fermat's and Euler's theorems. Also write down the applications of the two theorems. Moreover, write down the applications of linear congruences.

Q 10 State and prove Remainder Theorem and Factor Theorem. Also diminish the roots of $x^3 - 3x^2 + 5x + 9 = 0$ by 25.