Government Degree College, Baramulla (Autonomous)

Term End External Examination 4th Semester (Session- July 2024)

Subject: Mathematics

Course No and Title: MMTC1422M/Real Analysis-I

Time: 2.15 hours **Max Marks:100** **Min. Marks:40**

Section A: Objective Type Questions

Q1. Choose the appropriate Answer: **(8x1.5=12)**

i. The infimum of a set is the smallest member of the set.
- **A** always **B** sometimes
- **C** both are same **D** none of the above

ii. The smallest and the greatest member of the set $\left\{\frac{1}{n}; n \in N\right\}$ is
- **A** 0, 1 **B** 0, does not exist
- **C** does not exist, 1 **D** does not exist, does not exist

iii. A convergent sequence has _______ limit point(s).
- **A** finite **B** infinite
- **C** unique **D** depending on the given sequence

iv. Every bounded sequence has a limit point.
- **A** true **B** false
- **C** cannot say **D** depending on the given sequence

v. A positive term series converges if and only if the sequence of its partial sums is
- **A** convergent **B** divergent
- **C** bounded above **D** bounded below

vi. A series is said to be an alternating series whose terms are
- **A** alternately positive and negative **B** only positive
- **C** only negative **D** nothing can be said

vii. For a bounded function to be Riemann integrable on a partition P over [a, b]
- **A** $sup. L(P,f) \geq inf. U(P,f)$ **B** $sup. L(P,f) \leq inf. U(P,f)$
- **C** $sup. L(P,f) = inf. U(P,f)$ **D** none of the above

viii. If P_1 and P_2 are two partitions, then their common refinement given by
- **A** $P_1 + P_2$ **B** $P_1 \cup P_2$
- **C** $P_1 \cap P_2$ **D** none of the above

Section-B: Descriptive Type Questions (Short Type)

Q2: Answer all the Questions **(8 x 4 =32)**

i. Define field structure of a set.

ii. For any real numbers x, a, ϵ, show that $|x - a| < \epsilon$ if and only if a-ϵ < x < a+ ϵ.

iii. Write a short not on 'Sequence'.

iv. Define limit superior and limit inferior of a sequence.

v. Show that the series $\Sigma\frac{1}{n}$ does not converge.

vi. Show that the necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim_{n\to\infty} u_n = 0$.

vii. Show that the function f defined by $f(x) = \begin{cases} 0, & when\ x\ is\ rational \\ 1, & when\ x\ is\ irrational \end{cases}$ is not integrable on any interval.

viii. For any two partitions P_1 and P_2, show that $L(P_1, f) \leq U(P_2, f)$.

Section – C: Descriptive Type Questions (Medium Type)

Answer all the questions: **(4 x 7=28)**

Q 3. State and Prove Triangle Inequality.

OR

State and prove Archimedean Property of a set.

Q 4. State and Prove Bolzano-Weierstrass theorem.

OR

State and Prove Nested Interval theorem.

Q 5. Test the behaviour of the series $\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

OR

Test the behaviour of the series $\Sigma\frac{n+1}{n^p}$

Q6. Define refinement of a partition. If P^* is a refinement of a partition P, then show that for a bounded function.

$L(P^*, f) \geq L(P, f)$

$U(P^*, f) \leq U(P, f)$

OR

Show that *(3x+1)* is integrable on [1, 2] and $\int_1^2 (3x+1)dx = \frac{11}{2}$

Section – D: Descriptive Type Questions (Long Type)

Answer any two of the following: (2 x 14=28)

Q 7. Prove that the set of Rational numbers is not order-complete.

8. State and Prove Cauchy's first theorem on limits.

Q 9. Show that a positive term series $\Sigma\frac{1}{n^p}$ is convergent if and only if p >1.

Q 10 If f1and f2 are two bounded and integrable functions on [a, b], then f = f1 + f2 is also integrable on [a, b], and $\int_a^b f_1 dx + \int_a^b f_2\, dx = \int_a^b f dx.$