Term End External Examination 4th Semester (Session- July 2024)

Subject: Physcis

Course No and Title: PHYC2422M/Classical Mechanics

Time: 2.15 hours Max Marks:100 Min. Marks:40

Section A: Objective Type Questions

Q1. Choose the appropriate Answer: (8x1.5=12)

i. The dynamics of a particle governed by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$$

describes

A an undamped simple harmonic oscillator

B a damped harmonic oscillator with a time varying damping factor

C a undamped harmonic oscillator with a time dependent frequency

D a free particle

ii. The Lagrangian of a particle of mass m moving in one dimension is given by

$$L = \frac{1}{2}m\dot{x}^2 - bx$$

where b is a positive constant. The coordinates of the particle $x(t)$ at time t is given by: (in following c_1 and c_2 are constants)

A $-\frac{b}{2m}t^2 + c_1t + c_2$ **B** $c_1t + c_2$

C $\frac{b}{2m}t^2 + c_2$ **D** $\frac{b}{2m}t^2 + c_1$

iii. A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$. Which component (s) of the angular momentum is/are constant (s) of motion?

A L_x, L_y and L_z **B** only L_x and L_y

C only L_z **D** none

iv. The Hamiltonian of a system with n degrees of freedom is given by $H(q_1, \dots, q_n; p_1, \dots, p_n; t)$ with an explicit dependence on the time t. Which of the following is correct?

A Different phase trajectories cannot intersect each other.

B Any initial volume element in phase space remains unchanged in magnitude under time evolution.

C The equations $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$ are not valid since H has explicit time dependence

D H always represents the total energy of the system and is a constant of the motion.

v. A canonical transformation relates the old coordinates (q, p) to new ones (Q, P) by the relation $Q = q^2$ and $P = \frac{p}{2q}$. The corresponding time independent generating functionis

A $\frac{P}{q^2}$ **B** q^2P

C $\frac{q^2}{P}$ **D** qP^2

vi. If the given transformations $Q = q^m \cos np$, $P = q^m \sin np$ are canonical. What will be the value of m and n?

A $m = 1, n = 2$ **B** $m = \frac{1}{2}, n = 2$

C $m = 2, n = \frac{1}{2}$ **D** $m = 2, n = 1$

vii. The Poisson bracket $\{|r|, |p|\}$ has the value

A $|\boldsymbol{r}||\boldsymbol{p}|$ **B** $\vec{r}.\vec{p}$

C $\hat{r}.\hat{p}$ **D** none

viii. Which physical quantity is conserved due to the symmetry properties of Poisson brackets in Hamiltonian mechanics?

A Momentum **B** Energy

C Angular momentum **D** None

Section-B: Descriptive Type Questions (Short Type)

Q2: Answer all the Questions (8 x 4 =32)

i. A bead is constrained to move on a circular wire of radius R. Write the constraint equation and identify whether it is holonomic or non-holonomic.

ii. A system of particles is bound by a gravitational potential. The total kinetic energy of the system is $T = 10^6$J. Using the Virial

theorem, find the total potential energy.

iii. A particle moves in a central potential $V(r)$. Show that the angular momentum is conserved.

iv. Write Hamilton's equations of motion in terms of Hamiltonian function.

v. Describe the principle of least action and its significance in classical mechanics.

vi. What are the conditions that a transformation must satisfy to be considered canonical?

vii. State and explain Poisson's theorem.

viii. State and explain Liouville's theorem in Hamiltonian mechanics.

SECTION – C: DESCRIPTIVE TYPE QUESTIONS (Medium Type)

Answer all the questions: **(4 x 7=28)**

Q3. State Noether's theorem and explain the connection between symmetries and conservation laws.

OR

Define constraints in a mechanical system and classify them into holonomic and non-holonomic constraints with examples.

Q4. Define the canonical momentum for a system described by the Lagrangian $L(q_i, \dot{q}_i, t)$. Given a Lagrangian $L = \frac{1}{2}m\dot{x}^2 - V(x)$, find the canonical momentum and the Hamiltonian.

OR

Obtain the Hamiltonian and Hamilton's equations of motion one dimensional harmonic oscillator with mass m and spring constant k.

Q5. Show directly that transformations

a) $Q = \log\left(\frac{1}{q}\sin p\right), P = q\cot p$

b) $Q = p\tan q, P = \log(\sin p)$

are canonical.

OR

A particle of mass m moves in a one-dimensional potential $V(x) = \frac{1}{2}kx^2$. Use the principle of least action to find the path of the particle if it moves from $x_0 = 0$ at $t = 0$ to $x_f = A$ at $t = T$.

Q6. Find the Poisson bracket of

$$\alpha p^2 + 2\beta pq + \gamma q^2$$

with the Hamiltonian

$$H = ap^2 + bq^2 + cp + dq + e$$

where $\alpha, \beta, \gamma, a, b, c, d$ and e are all constants.

OR

Evaluate the following Poisson brackets

a) $[J_x, P_y]$ b) $[J_x, J_y]$ c) $[J_x, P_x]$.

Section – C: Descriptive Type Questions (Medium Type)

Answer all the questions: **(4 x 7=28)**

Q 7. Derive lagrangian equation of motion using D'Alembert's principle.

Q 8. Compare and contrast the Newtonian, Lagrangian, and Hamiltonian formulations of classical mechanics. Discuss the advantages and disadvantages of each approach in analyzing physical systems.

Q 9. What is a canonical transformation? What is the condition for the transformation to be canonical? How it can be used to reduce the number of independent variables? Can a canonical transformation reveal symmetry in a physical system?

Q 10. Define the Jacobi identity for Poisson brackets and explain its significance in Hamiltonian mechanics. Provide a detailed proof of the Jacobi identity