Term End External Examination 4th Semester (Session- July 2024)

Subject: Physics

Course No and Title: PHYC3422M/Mathematical Physics

Time: 2.15 hours **Max Marks:100** **Min. Marks:40**

Section A: Objective Type Questions

Q1. Choose the appropriate Answer: **(8x1.5=12)**

i. Which of the following functions is continuous but not differentiable at x=0.

A $f(x) = x^2$ **B** $f(x) = |x|$
C $f(x) = sinx$ **D** $f(x) = e^x$

ii. The degree of the differential equation $y''' + 2(y'')^3 + 5y' = 0$ is

A 3 B 2
C 1 D 0

iii. What is the value of $e^{i\pi}$?

A 1 B -1
C *i* D -*i*

iv. If $f(z) = u(x,y) + i\,v(x,y)$, is an analytical function then $f'(z)$ equals

A $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ **B** $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$
C $\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}$ **D** None of the above

v. If A is a (n x n) identity matrix. What is the trace of A?

A 0 **B** 1
C n **D** n^2

vi. The eigen values of a matrix A are 2, 3, and 1. What is the value of |A|?

A 5 **B** 2
C 3 **D** 6

vii. The Fourier Series of an odd function will contain

A only sine terms **B** only cosine terms
C both sine and cosine terms **D** neither sine nor cosine terms

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viii. What is the Fourier Transform of delta function $\delta(x)$?

A 0 **B** 1
C $e^{i\omega}$ **D** $\delta(\omega)$

Section-B: Descriptive Type Questions (Short Type)

Q2: Answer all the Questions **(8 x 4 =32)**

i. Find $\frac{dy}{dx}$, if $y + siny = cosx$.

ii. Find the value of λ for which the function

$$f(x) = \begin{cases} \lambda x + 1, & x \le 3 \\ 3x + 4, & x > 3 \end{cases}$$

is continuous.

iii. State and prove De Moivre's Theorem.

iv. Using Cauchy – Rieman equations, show that the function $e^x(cosy + i\,siny)$ is an analytic function.

v. If A and B are invertible matrices of same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

vi. Show that $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ is unitary matrix.

vii. What are Dirichlet's Conditions for a Fourier Series?

viii. If $F(s)$ is the complex Fourier Transform of $f(x)$, then prove that

$$F\{f(x-a)\} = e^{isa}F(s)$$

Section – C: Descriptive Type Questions (Medium Type)

Answer all the questions: **(4 x 7=28)**

Q3. Plot the graph of the function $f(x) = |x|$ and discuss the continuity of the function.

OR

Solve the following differential equation.

$$\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$$

Q4. Prove that for two complex numbers Z_1 and Z_2, $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$.

OR

Solve $x^4 + 1 = 0$.

Q5. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew - symmetric matrix.

OR

Prove that the eigen values of a Hermitian matrix are all real.

Q6. Find the Fourier Transform of

$$f(t) = \begin{cases} t & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$$

OR

Deduce expansion for Fourier Series in the complex form.

Section – D: Descriptive Type Questions (Long Type)

Answer any two of the following: **(2 x 14=28)**

Q7. Show that the following differential equation is a Homogeneous equation and then solve it.

$$x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$$

Q8. Derive Cauchy – Rieman Conditions. Determine whether $1/z$ is analytic or not?

Q9. Find the eigen values and eigen vectors of

$$\begin{bmatrix} 8 & -6 & 0 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q10. Find the Fourier expansion of $f(x) = x^2$, and show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi}{6}$.